OPTIMAL BI-IMPULSIVE NON-COPLANAR MANEUVERS USING HYPERBOLIC ORBITAL TRANSFER WITH TIME CONSTRAINT

Evandro Marconi Rocco

Instituto Nacional de Pesquisas Espaciais – INPE
C.P. 515, 12245-970, São José dos Campos, SP, Brazil
evandro@dem.inpe.br

Antonio Fernando Bertachini de Almeida Prado

Instituto Nacional de Pesquisas Espaciais – INPE
C.P. 515, 12245-970, São José dos Campos, SP, Brazil
prado@dem.inpe.br

Marcelo Lopes de Oliveira e Souza

Instituto Nacional de Pesquisas Espaciais – INPE
C.P. 515, 12245-970, São José dos Campos, SP, Brazil
marcelo@dem.inpe.br

José Everardo Junior Baldo

Instituto Nacional de Pesquisas Espaciais – INPE
C.P. 515, 12245-970, São José dos Campos, SP, Brazil
mr_hyde.geo@yahoo.com

***Abstract**: In this work the problem of two-impulsive orbital transfers between non-coplanar circular or elliptical orbits is studied using hyperbolic orbits as the transfer orbits, with minimum fuel consumption but with time limit for this transfer. The equations presented by Eckel and Vinh (1984) was used. These equations provide the elliptical transfer orbit between non-coplanar elliptical orbits with minimum fuel and fixed time of transfer; or minimum time of transfer for a prescribed fuel consumption, using elliptic transfer orbits. But, in this work, only the problem with minimum fuel consumption and fixed time of transfer was considered. Then, the equations presented by Eckel and Vinh (1984) was adapted to consider the problem of non-coplanar orbital transfer between circular and elliptical orbits using hyperbolic orbits as the transfer orbit and we developed a software for orbital maneuvers. The original method, developed by Eckel and Vinh, was presented without numerical results in that paper. Thus, the modifications considering the maneuvers between circular orbits, the implementation for the hyperbolic case and the solutions using this method are contributions of this work. The software was tested simulating real maneuvers with success.*

1 Introduction

The majority of the spacecrafts that have been placed in orbit around the Earth utilizes the basic concept of orbital transfers. During the launch, the spacecraft is placed in a parking orbit distinct from the final orbit for which the spacecraft was designed. Therefore, to reach the desired final orbit the spacecraft must perform orbital transfers. Besides that, the spacecraft orbit must be corrected periodically because there are perturbations acting on the spacecraft. Both maneuvers are usually calculated with minimum fuel consumption but without a time constraint. This time constraint imposes a new characteristic to the problem that rules out the majority of the transfer methods available in the literature: Hohmann (1925), Hoelker et al. (1959), Gobetz et al. (1969), Prado (1989), etc. Therefore, the transfer methods must be adapted to this new constraint: Prussing et al. (1986), Eckel (1982), Eckel et al. (1984), Lawden (1993) and Taur et al. (1995). In Brazil, there are important applications with the launch of China Brazil Earth Resources Satellites CBERS.

In this work it was considered the problem of two-impulse orbital transfers between non-coplanar elliptical orbits using a hyperbolic orbit as the transfer orbit, with minimum fuel consumption but with time limit for this transfer. This problem is very important because in many missions it is important to perform the maneuvers in the minimum time, as for instance in the case of remote sensing satellites, because during the maneuver the collected data are of low quality and therefore they cannot be used.

2 Nomenclature

a	$\to$	Semi-major axis
e	$\to$	Eccentricity
p	$\to$	Semi-latus rectum
ω	$\to$	Longitude of the periapsis
i	$\to$	Inclination
Ω	$\to$	Longitude of the ascending node
M	$\to$	Mean anomaly
E	$\to$	Eccentric anomaly
λ	$\to$	Angle between the planes of the initial and final orbits
β_1	$\to$	True anomaly of the point N obtained in the plane of the initial orbit
β_2	$\to$	True anomaly of the point N obtained in the plane of the final orbit
I_1	$\to$	Location of the first impulse
I_2	$\to$	Location of the second impulse
Δ	$\to$	Transfer angle obtained in the plane of the transfer orbit
V_1	$\to$	Velocity increment generated by the first impulse
V_2	$\to$	Velocity increment generated by the second impulse
V	$\to$	Total velocity increment
T	$\to$	Time spent in the maneuver
α_1	$\to$	True anomaly of the point I_1 obtained in the plane of the initial orbit
α_2	$\to$	True anomaly of the point I_2 obtained in the plane of the final orbit
r_1	$\to$	Distance from point I_1
r_2	$\to$	Distance from point I_2
f_1	$\to$	True anomaly of the point I_1 obtained in the plane of the transfer orbit
f_2	$\to$	True anomaly of the point I_2 obtained in the plane of the transfer orbit
x_1	$\to$	Radial component of the first impulse
x_2	$\to$	Radial component of the second impulse
y_1	$\to$	Transverse component of the first impulse in the plane of the initial orbit
y_2	$\to$	Transverse component of the second impulse in the plane of the transfer orbit
z_1	$\to$	Component of the first impulse orthogonal to the initial orbit
z_2	$\to$	Component of the second impulse orthogonal to the transfer orbit
h_i	$\to$	Horizontal component of V_i
N	$\to$	Intersection of the orbits

3 Definition of the troblem

The orbital transfer of a spacecraft from an initial orbit to a desired final orbit consists (Marec 1979) in a change of state of the spacecraft, from initial conditions $\vec{r}_0$, $\vec{v}_0$ and m_0 at time t_0 to final conditions $\vec{r}_f$, $\vec{v}_f$ and m_f at time t_f ($t_f \geq t_0$). In this work, it was considered that the spacecraft propulsion system is able to apply an impulsive thrust. Therefore, we have the instantaneous variation of the spacecraft velocity.

4 Presentation of the method

The bases for this method are the equations presented by Eckel et al. (1984). These equations provide the transfer orbit between non-coplanar elliptical orbits with minimum fuel and fixed time of transfer, or the transfer orbit with minimum time transfer for prescribed fuel consumption. Here, it was considered the problem with minimum fuel consumption and fixed time of transfer considering non-coplanar maneuvers between elliptical orbits using a hyperbolic orbit as the transfer orbit. The problem with minimum fuel and fixed time of transfer has already been studied by Rocco (1997) and by Rocco et al. (1999, 2002) for the non-coplanar case using elliptical orbit as the transfer orbit. The case of orbital transfer between non-coplanar orbits using elliptical orbit as the transfer orbit, with minimum time for a prescribed fuel consumption was already studied in Rocco et al. (2000).

The equations were presented in the literature but the method was not implemented neither tested by Eckel and Vinh. They used the plane of the transfer orbit as the reference plane but in this work the equatorial plane was used as the reference plane because in this way it is easy to obtain and to apply the results in real applications. Thus, the reference system is changed, adding the Eqs. 1 to 6.

Given two terminal orbits the intention was to obtain a transfer orbit which performs an orbital maneuver from the initial orbit to the final orbit with minimum velocity increment and fixed time of transfer. The orbits are specified by their orbital elements, which are presented in the item 2.

From the geometry of the non-coplanar maneuver we obtain β_1, β_2, λ and the transfer angle Δ:

$$\beta_1 = \arctan\left[\frac{\sin(\Omega_2 - \Omega_1)\tan(180^\circ - i_2)}{\sin i_1 + \tan(180^\circ - i_2)\cos i_1 \cos(\Omega_2 - \Omega_1)}\right] - \omega_1 \quad (1)$$

$$\beta_2 = \arctan\left[\frac{sin(\Omega_2 - \Omega_1)\tan i_1}{sini_2 + \tan i_1 \cos(180^\circ - i_2)\cos(\Omega_2 - \Omega_1)}\right] - \omega_2 \quad (2)$$

$$\lambda = \arcsin\left[\frac{sin(\Omega_2 - \Omega_1)sini_1}{sin(\omega_2 + \beta 2)}\right] = \arcsin\left[\frac{sin(\Omega_2 - \Omega_1)sini_2}{sin(\omega_1 + \beta_1)}\right] \quad (3)$$

$$\cos\Delta = \cos(\beta_1 - \alpha_1)\cos(\alpha_2 - \beta_2) + sin(\beta_1 - \alpha_1)sin(\alpha_2 - \beta_2)\cos(180^\circ - \lambda) \quad (4)$$

$$sin\Delta = \frac{sin(\alpha_2 - \beta_2)sin(180^\circ - \lambda)}{sinB} \quad (5)$$

$$B = \arctan\left[\frac{sin(180^\circ - \lambda)}{sin(\beta_1 - \alpha_1)\cot(\alpha_2 - \beta_2) - \cos(\beta_1 - \alpha_1)\cos(180^\circ - \lambda)}\right] \quad (6)$$

The geometry of the hyperbolic maneuver is shown in Fig. 1.

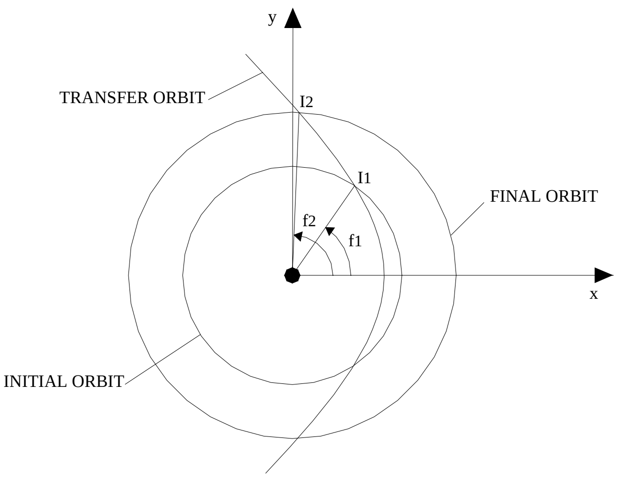

Figure 1. Hyperbolic Maneuver.

Considering that the maneuver is bi-impulsive, the total velocity increment is:

$$V = V_1 + V_2 = F(X) \quad (7)$$

The time of the transfer maneuver is:

$$T = G(X) \tag{8}$$

Therefore, the problem is the minimization of V for a prescribed T. If the time of transfer is prescribed, being equal to a value T_0, we have the constrained relation:

$$T - T_0 = 0 \tag{9}$$

Thus, we have the performance index:

$$J = V + k(T - T_0) \tag{10}$$

From Eckel et al. (1984) we know that the solution of the problem depends on three variables: the semi-latus rectum p of the transfer orbit and the true anomaly α_1 and α_2 that define the position of the impulses in the initial and final orbits. Therefore, we have the necessary conditions:

$$\frac{\partial V}{\partial p} + k\frac{\partial T}{\partial p} = 0 \quad ; \quad \frac{\partial V}{\partial \alpha_1} + k\frac{\partial T}{\partial \alpha_1} = 0 \quad ; \quad \frac{\partial V}{\partial \alpha_2} + k\frac{\partial T}{\partial \alpha_2} = 0 \tag{11}$$

By eliminating the Lagrange's multiplier k from Eqs. (11) we have the set of two equations:

$$\frac{\partial V}{\partial p}\frac{\partial T}{\partial \alpha_1} - \frac{\partial V}{\partial \alpha_1}\frac{\partial T}{\partial p} = 0 \quad ; \quad \frac{\partial V}{\partial p}\frac{\partial T}{\partial \alpha_2} - \frac{\partial V}{\partial \alpha_2}\frac{\partial T}{\partial p} = 0 \tag{12}$$

Evaluating the partial derivatives in these equations and doing some simplifications we have the final optimal conditions:

$$\begin{aligned}&(X_1 + YZes\inf_2)(S_1 q_1 - T_1 es\inf_1) + S_1 T_1 + W_1\left(\frac{W_1 - W_2}{sin\Delta}q_2 - W_1\tan\frac{\Delta}{2}\right)\\ &- \frac{W_1 Zer_1 e_1 sin\alpha_1}{q_1 p_1 s\inf_1\, sin\gamma_1} = 0\end{aligned} \tag{13}$$

$$\begin{aligned}&(X_2 + YZes\inf_1)(S_2 q_2 - T_2 es\inf_2) + S_2 T_2 - W_2\left(\frac{W_2 - W_1}{sin\Delta}q_1 - W_2\tan\frac{\Delta}{2}\right)\\ &+ \frac{W_2 Zer_2 e_2 sina_2}{q_2 p_2 s\inf_2\, sin\gamma_2} = 0\end{aligned} \tag{14}$$

Thus, we have a system of equations composed by Eqs. (9), (15) and (16). Solving this system of equations by using the Newton Raphson Method (cf. Press et al. 1992) or by using the Least Square Method (cf. Rocco 2002), we obtain the transfer orbit that performs the maneuver between two coplanar terminal orbits spending minimum fuel consumption, but with a specific time of transfer.

5 Results

Figures (2) to (8) present some results obtained with the software developed. They not only show the tendency of the parameters, but they quantify the evolution of the variables studied. The graphs were obtained through the variation of the time spent in the maneuver.

We utilized, as an example, the correction maneuver between two elliptical non-coplanar orbits where the initial orbit have the semi-major axis of 7000 km, eccentricity 0.015, inclination 0.001 rad, longitude of the periapsis 1.0 rad and longitude of ascending node 0.001 rad. The final orbit have the semi-major axis of 7729 km, eccentricity 0.015, inclination 0.0001 rad, longitude of the periapsis 1.0 rad and longitude of ascending node 0.01 rad. We used in this example the initial values p = 7000 km, α_1 = 3.71681469 rad, and α_2 = 6 rad. The graphs were obtained through the variation of the time spent in the maneuver from 7 to 82 s.

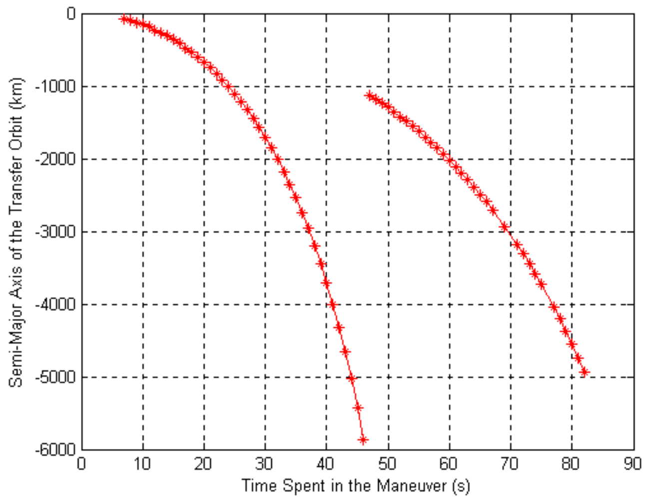

Figure 2. Semi-Major Axis vs. Time.

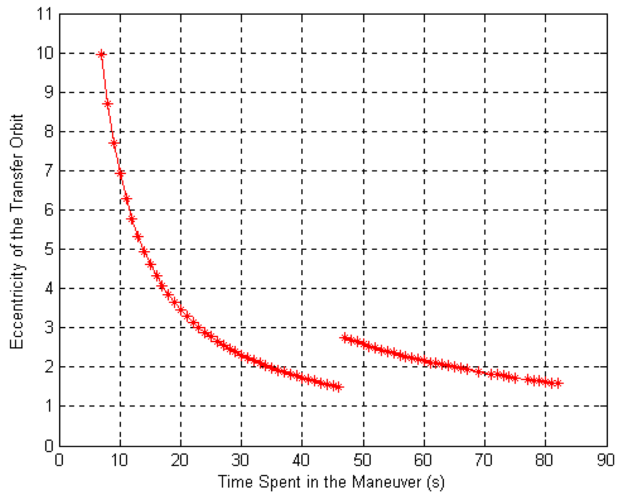

Figure 3. Eccentricity vs. Time.

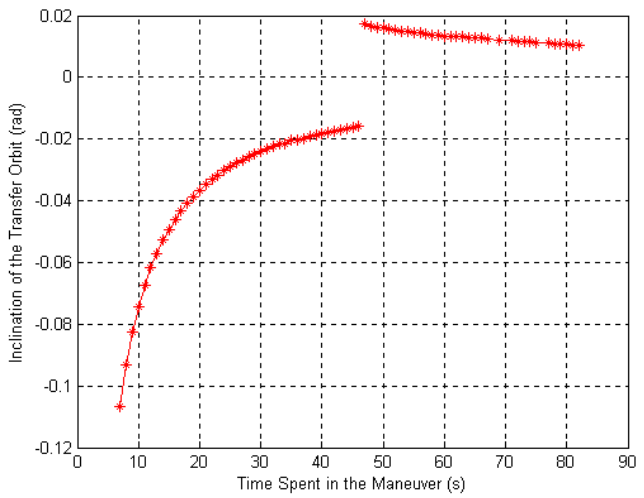

Figure 4. Inclination vs. Time.

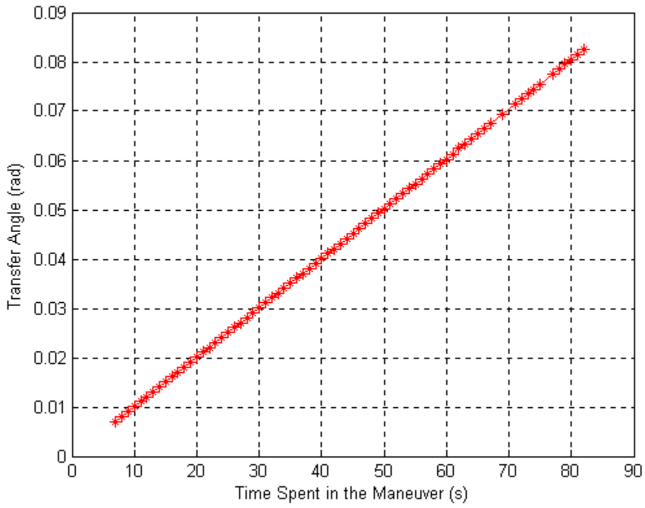

Figure 5. Transfer Angle vs. Time.

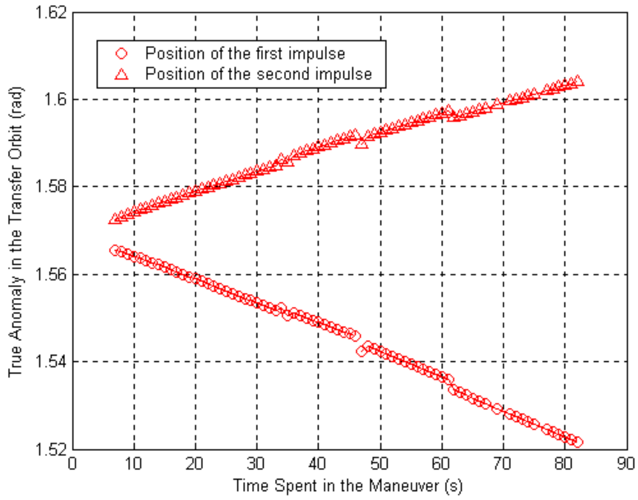

Figure 6. True Anomaly vs. Time.

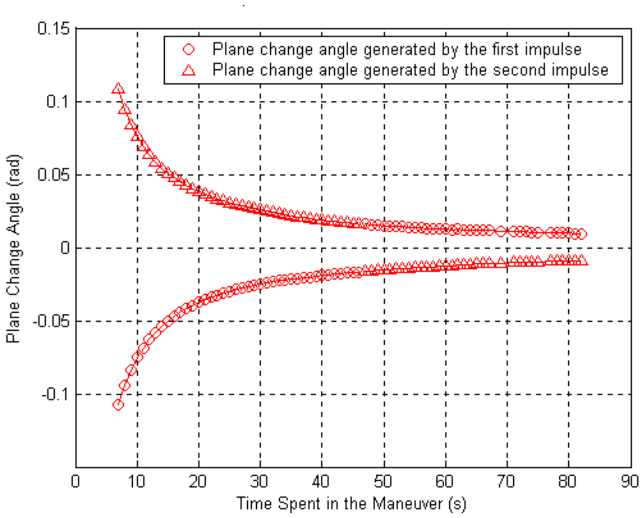

Figure 7. Plane Change Angle vs. Time.

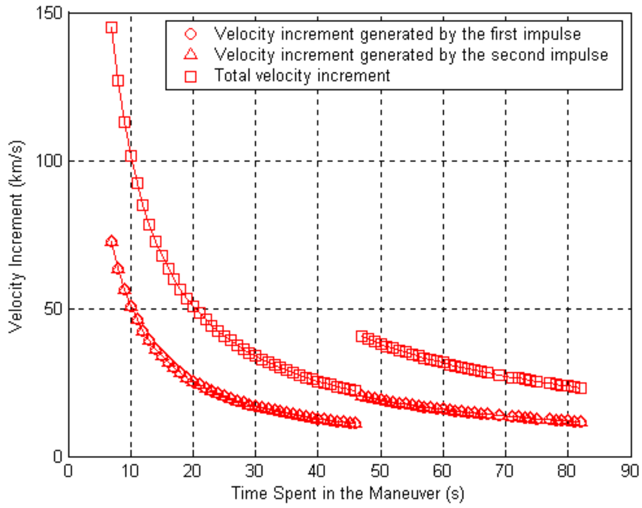

Figure 8. Velocity Increment vs. Time.

6 Conclusion

From Figs. (2) to (8) it is possible to verify the behavior of some orbital elements of the transfer orbit when the time spent in the maneuver was varied. In all figures it is possible to notice that there is a break point when the time is 47s. This break point is due to a change in the geometry of the maneuver in this point, as can be seen in Fig. (7), where there is an inversion of the plane change angles generated by the first and second impulses. In Fig. (2) we have the behavior of the semi-major axis. It may be verified that when the time is fixed in 7 *s* the semi-major axis becomes practically zero. When the time spent in the maneuver increases, the semi-major axis decreases. In Fig. (3) we have the behavior of the eccentricity. It may be verified that the eccentricity increases for a small value of time, as expected. In Fig. (4) it may be observed the inclination of the transfer orbit. In Fig. (5) it may be verified that the transfer angle decreases when the time is reduced. This can also be observed in Fig. (6), which presents the location of the impulses applied. In Fig. (7) it may be observed the plane change angle generated by the impulses. From this figure we conclude that the sum of the plane change angles almost remain constant because the second impulse undo part of the plane change angle that results of the first impulse. In Fig. (8) it may be verified that the velocity increment to accomplish the maneuver when the fixed time is 7 *s* is very large. When the time is increased, the necessary velocity increment decreases drastically because in this case the maneuver is accomplished by a large transfer angle, so the direction of the impulses approach the directions of the velocity vector of the satellite in the initial and final orbits. Thus, the fuel consumption becomes smaller, as it can be seen in the Fig. (9).

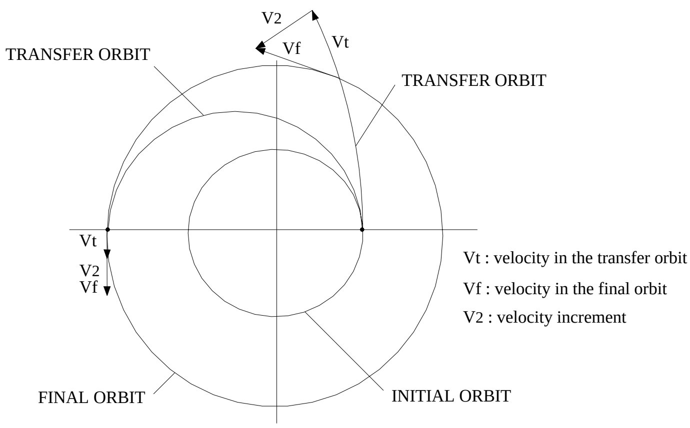

Figure 9. Impulses direction.

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