ORBITAL CHARACTERISTICS OF ARTIFICIAL SATELLITES IN RESONANCE AND THE CORRESPONDENT GEOPOTENCIAL COEFFICIENTS

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Abstract: *the purpose of this work is to present the orbital eccentricities and inclinations characteristics for some real artificial satellites, some of them already inactive, whose mean motions are commensurable with the Earth's rotation period. The correspondent geopotential coefficients for each considered resonance are also presented.*

1 Introduction

The influence of resonances in the translational and the rotational motion of artificial satellite has been studied covering several aspects such as:

a) considering commensurabilities of the satellite's orbital motion with the planet's rotational motion (see, for instance, including internal cited references, Gedeon et al., 1967; Sochilina, 1982; Grosso, 1980; Ely and Howell, 1996; Lima Jr., 2000; Klokocnik et al., 2003);
b) considering critical inclination (Allan, 1965; Gedeon, 1969; Delhaise and Henrard, 1991);
c) considering lunisolar perturbations (Cook, 1962; Hughes 1980; Breiter, 2001; Lima Jr et al, 2001; Deleflie et al. 2005),
d) including sun-syncronous orbits (Hough, 1981);
e) considering solar radiation pressure (Polyakova, 1963; Ferraz-Mello, 1979; Vilhena de Moraes, 1979; El-Saftawi, 2004);
f) considering spin-orbit coupling (Beletskii, 1975; Hamill and Blitzer, 1974; Vilhena de Moraes and Silva, 1990);
g) considering frequencies related with the rotational motion (Hitzl and Breakwell, 1970; Modi and Pand, 1975), and including vibrations of some parts of the satellites (Pringle, 1973; Cochran and Holloway, 1980).

The resonance considered here is the commensurability between n, the satellite's mean motion, and ω_E, the mean angular rate of rotation of the Earth, that is,

$$pn - q\omega_E \approx 0$$

where p and q are integers .

Of course, due to the non-uniform distribution of the Earth's mass, it must be also considered the precession of the angular keplerian elements.

The long period behaviour of the orbital elements, considering perturbations due to this resonance, is distinct for each pair (p,q) and for satellites in orbits of small or great eccentricities and or inclinations (Formiga, 2005). In this work, it is exhibited several actual examples of such satellites. The knowledge of real cases is important for the construction of theories with orbital maintenance or surveillance purposes when resonance is taking into account. Since the magnitude of the orbital perturbations due to a resonance depends on the harmonic coefficients, it is also presented here, for each considered resonance, the corresponding main harmonics coefficients.

2 Geopotential

The amplitudes of terms that produce the resonance are functions of the geopotential coefficients. The considered potential here is (Osório, 1973):

$$U = \frac{\mu}{2a} + \sum_{\ell=2}^{\infty}\sum_{m=0}^{\ell}\sum_{p=0}^{\ell}\sum_{q=-\infty}^{\infty}\frac{\mu}{a}\left(\frac{a_e}{a}\right)^{\ell} J_{\ell m} F_{\ell mp}(i) H_q^{-(\ell+1),(\ell-2p)}(e)\cos\varphi_{\ell mpq}(M,\omega,\Omega,\Theta) \quad (1)$$

where:

$$\varphi_{\ell mpq}(M,\omega,\Omega,\Theta) = qM + (\ell-2p)\omega + m(\Omega-\Theta-\lambda_{\ell m}) + (\ell-m)\frac{\pi}{2} \quad (2)$$

Here a, e, I, M, ω, Ω are the keplerian elements, $\Theta = \omega_E t$, J_{lm} are coefficients depending on the Earth's mass distribution, λ_{lm} is the longitude of the semi-major axis of symmetry for the harmonic (l,m); $F_{lmp}(I)$ represents the inclination functions. Since small and high eccentricities are considered, the Hansen's coefficients $H_q^{-(l-p),(l-2p)}(e)$ are used.

3 The satellites considered

The satellites considered in this paper were selected from a sample of 1818 satellites, chosen at random, extracted from the NORAD 2-line elements (CELESTRACK), whose orbital periods T_S satisfies the inequality

$$\left|pT_S - qT_E\right| \le 3\,\text{min} \quad (3)$$

where T_E is the Earth period of rotation.

The survey was taken from objects that are in orbit since the first release in 1957, some of them already inactive. From the selected sample, 1571 are close to a deep resonance.

In Table 1 we can find the number of satellites of the sample that are orbiting in a given resonance, as well as some orbital characteristics of the satellites in resonance, in agreement with its inclination (i) and eccentricity (***e***).

Table 1. Number of satellites in a given resonance.

resonance	Number of satellites	**resonance**	Number of satellites	**resonance**	Number of satellites
1:1		**10:1**		**14:1**	
e≤0,1 e i≤5°	330	*e*≤0,1 e 5°<i<55°	2	*e*≤0,1 e i≤5°	9
e≤0,1 e 5°<i<55°	51	*e*>0,1 e 5°<i<55°	1	*e*≤0,1 e i≥70°	528
e>0,1 e 5°<i<55°	1	**11:1**		*e*≤0,1 e 5°<i<70°	36
e>0,1 e 55°<i<70	6	*e*≤0,1 e 55°<i<70°	1	**15:1**	
2:1		*e*>0,1 e i≥70°	3	*e*≤0,1 e i≤5°	1
e≤0,1 e 55°<i<70°	16	*e*>0,1 e 5°<i<55°	2	*e*≤0,1 e i≥70°	290
e≤0,1 e 5°<i<55°	14	**12:1**		*e*≤0,1 e 5°<i<70°	41
e>0,1 e 5°<i<70°	25	*e*≤0,1 e i≥70°	22	*e*≤0,1 e 5°<i<55°	26
8:1		*e*≤0,1 e 5°<i<55°	3	**16:1**	
e≤0,1 e i≥70°	3	*e*>0,1 e i>=70°	9	*e*≤0,1 e i≤5°	1
9:1		*e*>0,1 e 5°<i<55°	5	*e*≤0,1 e i≥70°	29
e≤0,1 e i≥70°	1	**13:1**		*e*≤0,1 e 5°<i<70°	8
e≤0,1 e 5°<i<55°	1	*e*≤0,1 e i≤5°	2	*e*≤0,1 e 5°<i<55°	22
e>0,1 e 5°<i<55°	2	*e*≤0,1 e i≥70°	22	**13:2**	
17:8		*e*≤0,1 e 55°<i<70°	22	*e*≤0,1 e 5°<i<55°	2
e≤0,1 e 5°<i<70°	11			*e*≤0,1 e 5°<i<55°	2

4 Zonal and tesserals

The following main geopotential coefficients that are related with the corresponding resonance are explicitly given as follow:

Resonance 1:1

$$-\frac{\mu}{a}\{J_{2,1}\left(\frac{a_e}{a}\right)^2[H_1^{-3,2}(e)F_{2,1,0}(i)\operatorname{sen}(M-\Theta+2\omega+\Omega-\lambda_{2,1})+$$
$$H_1^{-3,0}(e)F_{2,1,1}(i)\operatorname{sen}(M-\Theta+\Omega-\lambda_{2,1})+$$
$$H_1^{-3,-2}(e)F_{2,1,2}(i)\operatorname{sen}(M-\Theta-2\omega+\Omega-\lambda_{3,1})]+$$
$$J_{3,1}\left(\frac{a_e}{a}\right)^3[H_1^{-4,3}(e)F_{3,1,0}(i)\cos(M-\Theta+3\omega+\Omega-\lambda_{3,1})+$$
$$H_1^{-4,1}(e)F_{3,1,1}(i)\cos(M-\Theta+\omega+\Omega-\lambda_{3,1})+$$
$$H_1^{-4,-1}(e)F_{3,1,2}(i)\cos(M-\Theta-\omega+\Omega-\lambda_{3,1})+$$
$$H_1^{-4,-3}(e)F_{3,1,3}(i)\cos(M-\Theta-3\omega+\Omega-\lambda_{3,1})]\}$$

Resonance 2:1

$$-\frac{\mu}{a}\{J_{2,2}\left(\frac{a_e}{a}\right)^2[H_1^{-3,2}(e)F_{2,2,0}(i)\cos(M+2\omega+2(\Omega-\Theta-\lambda_{2,2}))+$$
$$H_1^{-3,0}(e)F_{2,2,1}(i)\cos(M+2(\Omega-\Theta-\lambda_{2,2}))+$$
$$H_1^{-3,-2}(e)F_{2,2,0}(i)\cos(M-2\omega+2(\Omega-\Theta-\lambda_{2,2}))]+$$
$$J_{3,2}\left(\frac{a_e}{a}\right)^3[H_1^{-4,3}(e)F_{3,2,0}(i)\operatorname{sen}(M+3\omega+2(\Omega-\Theta-\lambda_{3,2}))+$$
$$H_1^{-4,1}(e)F_{3,2,1}(i)\operatorname{sen}(M+\omega+2(\Omega-\Theta-\lambda_{3,2}))+$$
$$H_1^{-4,-1}(e)F_{3,2,2}(i)\operatorname{sen}(M-\omega+2(\Omega-\Theta-\lambda_{3,2}))+$$
$$H_1^{-4,-3}(e)F_{3,2,3}(i)\operatorname{sen}(M-3\omega+2(\Omega-\Theta-\lambda_{3,2}))]\}$$

Resonance 3:1

$$\frac{\mu}{a}\{J_{3,3}\left(\frac{a_e}{a}\right)^3[H_1^{-4,3}(e)F_{3,3,0}(i)\cos(M+3\omega+3(\Omega-\Theta-\lambda_{3,3}))+$$
$$H_1^{-4,1}(e)F_{3,3,1}(i)\cos(M+\omega+3(\Omega-\Theta-\lambda_{3,3}))+$$
$$H_1^{-4,-1}(e)F_{3,3,2}(i)\cos(M-\omega+3(\Omega-\Theta-\lambda_{3,3}))+$$
$$H_1^{-4,-3}(e)F_{3,3,3}(i)\cos(M-3\omega+3(\Omega-\Theta-\lambda_{3,3}))]+$$
$$J_{4,3}\left(\frac{a_e}{a}\right)^4[H_1^{-5,4}(e)F_{4,3,0}(i)\operatorname{sen}(M+4\omega+3(\Omega-\Theta-\lambda_{4,3}))+$$
$$H_1^{-5,2}(e)F_{4,3,1}(i)\operatorname{sen}(M+2\omega+3(\Omega-\Theta-\lambda_{4,3}))+$$
$$H_1^{-5,0}(e)F_{4,3,2}(i)\operatorname{sen}(M+3(\Omega-\Theta-\lambda_{4,3}))+$$
$$H_1^{-5,-2}(e)F_{4,3,3}(i)\operatorname{sen}(M-2\omega+3(\Omega-\Theta-\lambda_{4,3}))]\}$$

Resonance 1:2

$$-\frac{\mu}{a}\{J_{2,1}\left(\frac{a_e}{a}\right)^2[H_1^{-3,2}(e)F_{2,1,0}(i)\,\mathrm{sen}(2M+2\omega+\Omega-\Theta-\lambda_{2,1})+$$
$$H_1^{-3,0}(e)F_{2,1,1}(i)\,\mathrm{sen}(2M+\Omega-\Theta-\lambda_{2,1})+$$
$$H_1^{-3,-2}(e)F_{2,1,2}(i)\,\mathrm{sen}(2M-2\omega+\Omega-\Theta-\lambda_{2,1})]+$$
$$J_{3,1}\left(\frac{a_e}{a}\right)^3[H_1^{-4,3}(e)F_{3,1,0}(i)\cos(2M+3\omega+\Omega-\Theta-\lambda_{3,1})+$$
$$H_1^{-4,1}(e)F_{3,1,1}(i)\cos(2M+\omega+\Omega-\Theta-\lambda_{3,1})+$$
$$H_1^{-4,-1}(e)F_{3,1,2}(i)\cos(2M-\omega+\Omega-\Theta-\lambda_{3,1})+$$
$$H_1^{-4,-3}(e)F_{3,1,3}(i)\cos(2M-3\omega+\Omega-\Theta-\lambda_{3,1})]\}$$

Resonance 8:1

$$\frac{\mu}{a}\{J_{9,8}\left(\frac{a_e}{a}\right)^9[H_1^{-10,9}(e)F_{9,8,0}(i)\,\mathrm{sen}(M+9\omega+8(\Omega-\Theta-\lambda_{9,8}))-$$
$$H_1^{-10,7}(e)F_{9,8,1}(i)\,\mathrm{sen}(M+7\omega+8(\Omega-\Theta-\lambda_{9,8}))-$$
$$H_1^{-10,5}(e)F_{9,8,2}(i)\,\mathrm{sen}(M+5\omega+8(\Omega-\Theta-\lambda_{9,8}))-$$
$$H_1^{-10,3}(e)F_{9,8,3}(i)\,\mathrm{sen}(M+3\omega+8(\Omega-\Theta-\lambda_{9,8}))-$$
$$H_1^{-10,1}(e)F_{9,8,4}(i)\,\mathrm{sen}(M+\omega+8(\Omega-\Theta-\lambda_{9,8}))-$$
$$H_1^{-10,-1}(e)F_{9,8,5}(i)\,\mathrm{sen}(M-\omega+8(\Omega-\Theta-\lambda_{9,8}))-$$
$$H_1^{-10,-3}(e)F_{9,8,6}(i)\,\mathrm{sen}(M-3\omega+8(\Omega-\Theta-\lambda_{9,8}))]+$$
$$J_{10,8}\left(\frac{a_e}{a}\right)^{10}[H_1^{-11,10}(e)F_{10,8,0}(i)\cos(M+10\omega+8(\Omega-\Theta-\lambda_{10,8}))-$$
$$H_1^{-11,8}(e)F_{10,8,1}(i)\cos(M+8\omega+8(\Omega-\Theta-\lambda_{10,8}))-$$
$$H_1^{-11,6}(e)F_{10,8,2}(i)\cos(M+6\omega+8(\Omega-\Theta-\lambda_{10,8}))-$$
$$H_1^{-11,4}(e)F_{10,8,3}(i)\cos(M+4\omega+8(\Omega-\Theta-\lambda_{10,8}))-$$
$$H_1^{-10,2}(e)F_{10,8,4}(i)\cos(M+2\omega+8(\Omega-\Theta-\lambda_{10,8})-$$
$$H_1^{-11,0}(e)F_{10,8,5}(i)\cos(M+8(\Omega-\Theta-\lambda_{10,8}))-$$
$$H_1^{-11,-2}(e)F_{10,8,6}(i)\cos(M-2\omega+8(\Omega-\Theta-\lambda_{10,8}))]\}$$

Resonance 9:1

$$\frac{\mu}{a} J_{9,9}\left(\frac{a_e}{a}\right)^9 [H_1^{-10,9}(e)F_{9,9,0}(i)\cos(M+9\omega+9(\Omega-\Theta-\lambda_{9,9}))+$$
$$H_1^{-10,7}(e)F_{9,9,1}(i)\cos(M+7\omega+9(\Omega-\Theta-\lambda_{9,9}))+$$
$$H_1^{-10,5}(e)F_{9,9,2}(i)\cos(M+5\omega+9(\Omega-\Theta-\lambda_{9,9}))+$$
$$H_1^{-10,3}(e)F_{9,9,3}(i)\cos(M+3\omega+9(\Omega-\Theta-\lambda_{9,9}))+$$
$$H_1^{-10,1}(e)F_{9,9,4}(i)\cos(M+\omega+9(\Omega-\Theta-\lambda_{9,9}))+$$
$$H_1^{-10,-1}(e)F_{9,9,5}(i)\cos(M-\omega+9(\Omega-\Theta-\lambda_{9,9}))+$$
$$H_1^{-10,-3}(e)F_{9,9,6}(i)\cos(M-3\omega+9(\Omega-\Theta-\lambda_{9,9}))+$$
$$H_1^{-10,-5}(e)F_{9,9,7}(i)\cos(M-5\omega+9(\Omega-\Theta-\lambda_{9,9}))+$$
$$H_1^{-10,-7}(e)F_{9,9,8}(i)\cos(M-7\omega+9(\Omega-\Theta-\lambda_{9,9}))+$$
$$H_1^{-10,-9}(e)F_{9,9,9}(i)\cos(M-9\omega+9(\Omega-\Theta-\lambda_{9,9}))]$$

Resonance 10:1

$$\frac{\mu}{a} J_{10,10}\left(\frac{a_e}{a}\right)^{10} [-H_1^{-11,10}(e)F_{10,10,0}(i)\cos(M+10\omega+10(\Omega-\Theta-\lambda_{10,10}))+$$
$$H_1^{-11,8}(e)F_{10,10,1}(i)\cos(M+8\omega+10(\Omega-\Theta-\lambda_{10,10}))+$$
$$H_1^{-11,6}(e)F_{10,10,2}(i)\cos(M+6\omega+10(\Omega-\Theta-\lambda_{10,10}))+$$
$$H_1^{-11,4}(e)F_{10,10,3}(i)\cos(M+4\omega+10(\Omega-\Theta-\lambda_{10,10}))+$$
$$H_1^{-11,2}(e)F_{10,10,4}(i)\cos(M+2\omega+10(\Omega-\Theta-\lambda_{10,10}))+$$
$$H_1^{-11,0}(e)F_{10,10,5}(i)\cos(M+10(\Omega-\Theta-\lambda_{10,10}))+$$
$$H_1^{-11,-2}(e)F_{10,10,6}(i)\cos(M-2\omega+10(\Omega-\Theta-\lambda_{10,10}))+$$
$$H_1^{-11,-4}(e)F_{10,10,7}(i)\cos(M-4\omega+10(\Omega-\Theta-\lambda_{10,10}))+$$
$$H_1^{-11,-6}(e)F_{10,10,8}(i)\cos(M-6\omega+10(\Omega-\Theta-\lambda_{10,10}))+$$
$$H_1^{-11,-8}(e)F_{10,10,9}(i)\cos(M-8\omega+10(\Omega-\Theta-\lambda_{10,10}))+$$
$$H_1^{-11,-10}(e)F_{10,10,10}(i)\cos(M-10\omega+10(\Omega-\Theta-\lambda_{10,10}))]$$

Resonance 11:1

$$\frac{\mu}{a} J_{11,11}\left(\frac{a_e}{a}\right)^{11} [H_1^{-12,11}(e)F_{11,11,0}(i)\cos(M+11\omega+11(\Omega-\Theta-\lambda_{11,11}))+$$
$$H_1^{-12,9}(e)F_{11,11,1}(i)\cos(M+9\omega+11(\Omega-\Theta-\lambda_{11,11}))+$$
$$H_1^{-12,7}(e)F_{11,11,2}(i)\cos(M+7\omega+11(\Omega-\Theta-\lambda_{11,11}))+$$
$$H_1^{-12,5}(e)F_{11,11,3}(i)\cos(M+5\omega+11(\Omega-\Theta-\lambda_{11,11}))+$$
$$H_1^{-12,3}(e)F_{11,11,4}(i)\cos(M+3\omega+11(\Omega-\Theta-\lambda_{11,11}))+$$
$$H_1^{-12,1}(e)F_{11,11,5}(i)\cos(M+\omega+11(\Omega-\Theta-\lambda_{11,11}))+$$
$$H_1^{-12,-1}(e)F_{11,11,6}(i)\cos(M-\omega+11(\Omega-\Theta-\lambda_{11,11}))+$$
$$H_1^{-12,-3}(e)F_{11,11,7}(i)\cos(M-3\omega+11(\Omega-\Theta-\lambda_{11,11}))+$$
$$H_1^{-12,-5}(e)F_{11,11,8}(i)\cos(M-5\omega+11(\Omega-\Theta-\lambda_{11,11}))+$$
$$H_1^{-12,-7}(e)F_{11,11,9}(i)\cos(M-7\omega+11(\Omega-\Theta-\lambda_{11,11}))+$$
$$H_1^{-12,-9}(e)F_{11,11,10}(i)\cos(M-9\omega+11(\Omega-\Theta-\lambda_{11,11}))+$$
$$H_1^{-12,-11}(e)F_{11,11,11}(i)\cos(M-11\omega+11(\Omega-\Theta-\lambda_{11,11}))]$$

Resonance 12:1

$$\frac{\mu}{a} J_{12,12}\left(\frac{a_e}{a}\right)^{12} [H_1^{-13,12}(e)F_{12,12,0}(i)\cos(M+12\omega+12(\Omega-\Theta-\lambda_{12,12}))+$$
$$H_1^{-13,10}(e)F_{12,12,1}(i)\cos(M+10\omega+12(\Omega-\Theta-\lambda_{12,12}))+$$
$$H_1^{-13,8}(e)F_{12,12,2}(i)\cos(M+8\omega+12(\Omega-\Theta-\lambda_{12,12}))+$$
$$H_1^{-13,6}(e)F_{12,12,3}(i)\cos(M+6\omega+12(\Omega-\Theta-\lambda_{12,12}))+$$
$$H_1^{-13,4}(e)F_{12,12,4}(i)\cos(M+4\omega+12(\Omega-\Theta-\lambda_{12,12}))+$$
$$H_1^{-13,2}(e)F_{12,12,5}(i)\cos(M+2\omega+12(\Omega-\Theta-\lambda_{12,12}))+$$
$$H_1^{-13,0}(e)F_{12,12,6}(i)\cos(M+12(\Omega-\Theta-\lambda_{12,12}))+$$
$$H_1^{-13,-2}(e)F_{12,12,7}(i)\cos(M-2\omega+12(\Omega-\Theta-\lambda_{12,12}))+$$
$$H_1^{-13,-4}(e)F_{12,12,8}(i)\cos(M-4\omega+12(\Omega-\Theta-\lambda_{12,12}))+$$
$$H_1^{-13,-6}(e)F_{12,12,9}(i)\cos(M-6\omega+12(\Omega-\Theta-\lambda_{12,12}))+$$
$$H_1^{-13,-8}(e)F_{12,12,10}(i)\cos(M-8\omega+12(\Omega-\Theta-\lambda_{12,12}))+$$
$$H_1^{-13,-10}(e)F_{12,12,11}(i)\cos(M-10\omega+12(\Omega-\Theta-\lambda_{12,12}))]$$

Resonance 13:1

$$\frac{\mu}{a} J_{13,13}\left(\frac{a_e}{a}\right)^{13} [H_1^{-14,13}(e)F_{13,13,0}(i)\cos(M+13\omega+13(\Omega-\Theta-\lambda_{13,13}))+$$
$$H_1^{-14,11}(e)F_{13,13,1}(i)\cos(M+11\omega+13(\Omega-\Theta-\lambda_{13,13}))+$$
$$H_1^{-14,9}(e)F_{13,13,2}(i)\cos(M+9\omega+13(\Omega-\Theta-\lambda_{13,13}))+$$
$$H_1^{-14,7}(e)F_{13,13,3}(i)\cos(M+7\omega+13(\Omega-\Theta-\lambda_{13,13}))+$$
$$H_1^{-14,5}(e)F_{13,13,4}(i)\cos(M+5\omega+13(\Omega-\Theta-\lambda_{13,13}))+$$
$$H_1^{-14,3}(e)F_{13,13,5}(i)\cos(M+3\omega+13(\Omega-\Theta-\lambda_{13,13}))+$$
$$H_1^{-14,1}(e)F_{13,13,6}(i)\cos(M+\omega+13(\Omega-\Theta-\lambda_{13,13}))+$$
$$H_1^{-14,-1}(e)F_{13,13,7}(i)\cos(M-\omega+13(\Omega-\Theta-\lambda_{13,13}))+$$
$$H_1^{-14,-3}(e)F_{13,13,8}(i)\cos(M-3\omega+13(\Omega-\Theta-\lambda_{13,13}))+$$
$$H_1^{-14,-5}(e)F_{13,13,9}(i)\cos(M-5\omega+13(\Omega-\Theta-\lambda_{13,13}))+$$
$$H_1^{-14,-7}(e)F_{13,13,10}(i)\cos(M-7\omega+13(\Omega-\Theta-\lambda_{13,13}))+$$
$$H_1^{-14,-9}(e)F_{13,13,11}(i)\cos(M-9\omega+13(\Omega-\Theta-\lambda_{13,13}))]$$

Resonance 15:1

$$\frac{\mu}{a} J_{15,15}\left(\frac{a_e}{a}\right)^{15} [H_1^{-16,15}(e)F_{15,15,0}(i)\cos(M+15\omega+15(\Omega-\Theta-\lambda_{15,15}))+$$
$$H_1^{-16,13}(e)F_{15,15,1}(i)\mathrm{sen}(M+13\omega+15(\Omega-\Theta-\lambda_{15,15}))+$$
$$H_1^{-16,11}(e)F_{15,15,2}(i)\mathrm{sen}(M+11\omega+15(\Omega-\Theta-\lambda_{15,15}))+$$
$$H_1^{-16,9}(e)F_{15,15,3}(i)\mathrm{sen}(M+9\omega+15(\Omega-\Theta-\lambda_{15,15}))+$$
$$H_1^{-16,7}(e)F_{15,15,4}(i)\mathrm{sen}(M+7\omega+15(\Omega-\Theta-\lambda_{15,15}))+$$
$$H_1^{-16,5}(e)F_{15,15,5}(i)\mathrm{sen}(M+5\omega+15(\Omega-\Theta-\lambda_{15,15}))+$$
$$H_1^{-16,3}(e)F_{15,15,6}(i)\mathrm{sen}(M+3\omega+15(\Omega-\Theta-\lambda_{15,15}))+$$
$$H_1^{-16,1}(e)F_{15,15,7}(i)\mathrm{sen}(M+\omega+15(\Omega-\Theta-\lambda_{15,15}))+$$
$$H_1^{-16,-1}(e)F_{15,14,8}(i)\mathrm{sen}(M-\omega+15(\Omega-\Theta-\lambda_{15,15}))+$$
$$H_1^{-16,-3}(e)F_{15,15,9}(i)\mathrm{sen}(M-3\omega+15(\Omega-\Theta-\lambda_{15,15}))+$$
$$H_1^{-16,-5}(e)F_{15,15,10}(i)\mathrm{sen}(M-5\omega+15(\Omega-\Theta-\lambda_{15,15}))+$$
$$H_1^{-16,-7}(e)F_{15,15,11}(i)\mathrm{sen}(M-7\omega+15(\Omega-\Theta-\lambda_{15,15}))]$$

Resonance 13:2

$$-\frac{\mu}{a}J_{15,13}\left(\frac{a_e}{a}\right)^{15}[H_2^{-16,15}(e)F_{15,13,0}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,13}(e)F_{15,13,1}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,11}(e)F_{15,13,2}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,9}(e)F_{15,13,3}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,7}(e)F_{15,13,4}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,5}(e)F_{15,13,5}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,3}(e)F_{15,13,6}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,1}(e)F_{15,13,7}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,-1}(e)F_{15,13,8}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,-3}(e)F_{15,13,9}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,-5}(e)F_{15,13,10}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,-7}(e)F_{15,13,11}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,-9}(e)F_{15,13,12}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,-11}(e)F_{15,13,13}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,-13}(e)F_{15,13,14}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))+$$
$$H_2^{-16,-15}(e)F_{15,13,15}(i)\cos(2M-17\omega-13(\Omega-\Theta-\lambda_{15,13}))]$$

Resonance 17:8

$$\frac{\mu}{a} J_{17,17}\left(\frac{a_e}{a}\right)^{17} [H_1^{-18,17}(e)F_{17,17,0}(i)\cos(8M - 17\omega - 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_8^{-18,15}(e)F_{17,17,1}(i)\cos(8M + 15\omega + 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_8^{-18,13}(e)F_{17,17,2}(i)\cos(8M + 13\omega - 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_8^{-18,11}(e)F_{17,17,3}(i)\cos(8M + 11\omega + 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_8^{-18,9}(e)F_{15,17,4}(i)\cos(8M + 9\omega - 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_8^{-18,7}(e)F_{17,17,5}(i)\cos(8M + 7\omega + 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_8^{-18,5}(e)F_{17,17,6}(i)\cos(8M + 5\omega - 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_8^{-18,3}(e)F_{17,17,7}(i)\cos(8M + 3\omega + 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_1^{-18,1}(e)F_{17,17,8}(i)\cos(8M + \omega - 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_8^{-18,-1}(e)F_{17,17,9}(i)\cos(8M - \omega + 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_1^{-18,-3}(e)F_{17,17,10}(i)\cos(8M - 3\omega - 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_8^{-18,-5}(e)F_{17,17,11}(i)\cos(8M - 5\omega - 17(\Omega - \Theta - \lambda_{17,17}))$$

$$H_8^{-18,-7}(e)F_{17,17,12}(i)\cos(8M - 7\omega + 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_8^{-18,-9}(e)F_{17,17,13}(i)\cos(8M - 9\omega - 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_8^{-18,-11}(e)F_{17,17,14}(i)\cos(8M - 11\omega + 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_8^{-18,-13}(e)F_{17,17,15}(i)\cos(8M - 13\omega - 17(\Omega - \Theta - \lambda_{17,17})) +$$

$$H_8^{-18,-15}(e)F_{17,17,16}(i)\cos(8M - 15\omega + 17(\Omega - \Theta - \lambda_{17,17}))]$$

5 Conclusions

The orbital characteristics and geopotential coefficients for some artificial satellites orbiting in resonance were presented. This can be useful for the study of the influences of the eccentricity and inclination in the orbital perturbations due to resonances.

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7 References

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