QUASI OPTIMAL SPACECRAFT FORMATION MANEUVERING VIA LYAPUNOV FUNCTIONS

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Abstract: *space mission concepts based on satellite formations feature several interesting and demanding design requirements. During formation deployment, re-sizing and re-orientation, the spacecraft must reach their desired positions without incurring in collisions or interfering with each other. Conventional GNC algorithms cannot support the degree of flexibility, the accuracy in the relative positioning and the fine pointing attitude requirements of future missions. The potential function method, based upon Lyapunov's second theorem on stability, brings the advantage of robustness and flexibility, along with a light workload for the control system. The aim of this paper is to foster the understanding of existing algorithms based on this method. In particular, the attention is focused on reducing the fuel consumption through the parametric optimization of the Lyapunov function that defines the control algorithm.*

1 Introduction

The issues related to the control of satellite formation have been challenging traditional control methods since the very beginning. For small formations in close proximity of the Earth, controlling each satellite separately from the ground station is a viable option. As the number of satellites within the formation and the distance of the operational orbit from the Earth increase, conventional methods show their limits and become less practical. New control methods are therefore required, approaches that enhance the automation of the system, enabling the formation to perform deployment, maintenance and re-configuration maneuvers autonomously. Open-loop control approaches have been proposed (Schaub *et al.*, 1999) and although simpler in their definition, depend heavily on spacecraft parameters uncertainties, knowledge of perturbation forces and disturbances within the control loop. Closed-loop control laws thus appear as the most viable option. Soft computing methods, although allowing a successful control through real time on-board navigation, are difficult to validate; moreover a significant software implementation is required. Considerations such as collision avoidance and constrained pointing directions render the planning of these operations time consuming, particularly if frequent payload re-pointing is necessary. Potential function methods, arising from Lyapunov's Second Method for stability analysis (Kalman and Bertram, 1960a; 1960b) have found extensive use in space applications (McInnes, 1994; 1995a; 1995b; Radice, 2000; 2006). This paper aims to improve the performance of the control algorithm in terms of fuel consumption by optimizing the parameters present in the artificial potential function. It must be noted that the optimization is not trivial, due to the complexity and nonlinearities of the dynamical system under consideration. Standard gradient-based optimization techniques are not useful for this case, even though a rapid convergence may be achieved they have to be initialized with a state close to the solution itself and the minimum may be only local. Due to the high nonlinearities of the system, finding such initial state for all parameters is too complex. Evolutionary techniques give better result but are time consuming, thus negating one of the strong points of the potential function method: the light computational workload. For these reasons a parametric optimization is performed and the results obtained are promising.

2 The potential function method

The potential function method is an extension of Lyapunov's second method and has found several applications in the field of space systems engineering. Given a set of differential equations $\dot{\mathbf{y}} = \mathbf{f}(\mathbf{x})$, which describe the time evolution of a dynamic system, and defining the desired final state, the convergence to this state and the global stability of the system can be guaranteed by building a Lyapunov potential function that has a global minimum in the final state. In fact, a function $V = V(\mathbf{x})$ is a Lyapunov function for the system if the following conditions are met:

1) $V(\bar{\mathbf{x}}) = 0$ (1a)

2) $V(\mathbf{x}) > 0 \quad for\ \mathbf{x} \neq \bar{\mathbf{x}}$ (1b)

3) $V(\mathbf{x}) \to \infty \quad for\ \|\mathbf{x}\| \to \infty$ (1c)

$$4)\ \dot{V}(\mathbf{x}) \le 0 \quad \textit{for}\ \mathbf{x} \ne \bar{\mathbf{x}} \tag{1d}$$

for goal state $\bar{\mathbf{x}}$. Lyapunov's theorem guarantees that a system that admits such a function is globally stable. The third condition of the theorem represents the mechanisms that drives the convergence of the state vector to the goal state. By differentiating the potential function with respect to time yields:

$$\dot{V} = \nabla f \cdot \dot{\boldsymbol{x}} \tag{2}$$

Therefore, by analytically determining the expression of the time derivative of the potential function, $\dot{V}$, the control required to render it negative can be found. Alternatively an expression for the velocity vector, $\dot{\boldsymbol{x}}$, can be found to fulfill the third condition of the theorem. In this case $\dot{\boldsymbol{x}}$ becomes a desired velocity, a possible definition of which is:

$$\mathbf{v}_{\text{desired}} = -k \frac{\nabla V}{\|\nabla V\|} \tag{3}$$

where $\nabla V/\|\nabla V\|$ is the unit vector parallel to the gradient of V, and k is a positive definite shaping parameter. In this way the potential derivative is forced to be non-positive by the actuators, providing the necessary action to match the current velocity of the spacecraft with the desired velocity, driving the system to the desired state. The most general expression for the potential function can be written as:

$$V = V_{\text{attractive}} + V_{\text{repulsive}} \tag{4}$$

The attractive component is a function of the distance, linear and angular, from the current spacecraft state to the desired goal state. The repulsive component accounts for any obstacles that have to be avoided during the maneuver; as the spacecraft approaches an obstacle the value of the repulsive component increases greatly thus modifying the gradient of the potential. This activates the control actuators and the spacecraft is guided towards a new direction avoiding any collision or attitude constraints.

3 Formation flight dynamics

We will now introduce the model used for the formation dynamics. The spacecraft are in the proximity of the Sun-Earth L2 point. Under the assumptions of the Restricted Circular Three Body Problem, the motion of the spacecraft around L2 can be accurately described by the following set of linearized equations (Alfriend *et al.*, 2002; Hamilton *et al.*, 2002):

$$\begin{aligned} &\ddot{x} - 2\omega\dot{z} - \left(\omega^2 + 2\mu_0^2\right)x = f_x \\ &\ddot{y} + \mu_0^2 y = f_y \\ &\ddot{z} + 2\omega\dot{x} + \left(\mu_0^2 - \omega^2\right)x = f_z \end{aligned} \tag{5}$$

where ω is the angular velocity of the system, $\mathbf{f} = f(f_x, f_y, f_z)$ are the external forces acting on the spacecraft and

$$\mu_0^2 = \frac{\mu_S}{R_{SL2}^3} + \frac{\mu_E}{R_{EL2}^3} \tag{6}$$

where μ_E and μ_S are respectively the Earth and the Sun gravitational constants and R_{EL2} and R_{SL2} are respectively the distance of the Earth and the Sun from the Sun-Earth L2 point. Equation (5) will be used to model the motion of each satellite in the formation.

4 Paramteric analysis

In this work the attention is focused primarily on the fuel consumption of a single satellite rather than of the entire formation. By looking at the general expression of the control algorithm, introduced by Eq. (3), we can see that the actuators will force the spacecraft to align its current velocity vector to that of the desired velocity. To improve the performances of the control algorithm we must therefore try to accurately shape both the amplitude and the direction of $\mathbf{v}_{\text{desired}}$. The way to achieve this is through the shaping parameter k:

$$k = \mu\left(1 - e^{-\lambda V}\right) \tag{7}$$

where μ and λ are shaping constants. These constants allow the shape of *k* to be modified; μ scales the amplitude and therefore the maximum velocity of the maneuver, while λ influences the deceleration phase, setting how hard the breaking action will be and how far from the final goal position it will start at. In general terms, the main problem that hinders fuel consumption is the abrupt beginning of the maneuver. It is apparent therefore that the efforts should be directed towards a careful and smart shaping of the three different aspects of shaping factor *k*: the start of the maneuver, the acceleration profile and the deceleration phase to the final position. The first step focuses on avoiding the abrupt initial acceleration, as the actuators receive the command from the control algorithm. By performing a polynomial regression it is possible to determine an expression of *k* as a function of the initial velocity:

$$k = \mu \frac{0.001 + 1.5745 v_{initial} + v_{initial}^2}{\mu\left(1 - e^{-\lambda}\right)} \left(1 - e^{-\frac{\lambda V}{V_0}}\right) \tag{8}$$

where V_0 is the initial value of the potential function. If the initial velocity is not zero, the satellite will accelerate until its velocity is equal to the desired one. If, on the other hand, the initial velocity is zero, or close to zero, Eq. (8) would give a constant magnitude, close to zero, for the desired velocity, thus preventing an effective action. A further modification is therefore introduced to ensure the spacecraft produces an adequate initial acceleration from static initial conditions:

$$k = \mu \left[\frac{\sqrt[3]{\left\| V_{ta} - V_{ta0} \right\|}}{V_{ta0}} + \frac{0.001 + 1.5745 v_{initial} + v_{initial}^2}{\mu\left(1 - e^{-\lambda}\right)} \right] \left(1 - e^{-\frac{\lambda V}{V_0}}\right) \tag{9}$$

where V_{ta} is the translation attractive component of the potential function and V_{ta0} is its initial value. Equation (9) allows a smooth start of the maneuver and provides a smooth acceleration profile. One final issue has however to be addressed: the difference between the maximum desired velocity and the initial velocity is constant. A desirable feature would be one that scales this difference as the initial velocity increases, as there would be little point in accelerating the spacecraft further if it already has a sufficiently high initial velocity. This feature can be implemented through parameter μ:

$$\mu = \mu_0 e^{-\alpha \left\| v_0 \right\|} \tag{10}$$

where α and μ_o are user defined parameters. Figure 1 shows the evolution of parameter *k*, as expressed in Eq. (9), for different initial velocities. It can be noted how the initial acceleration imparted upon the spacecraft is reduced as the initial velocity is increased.

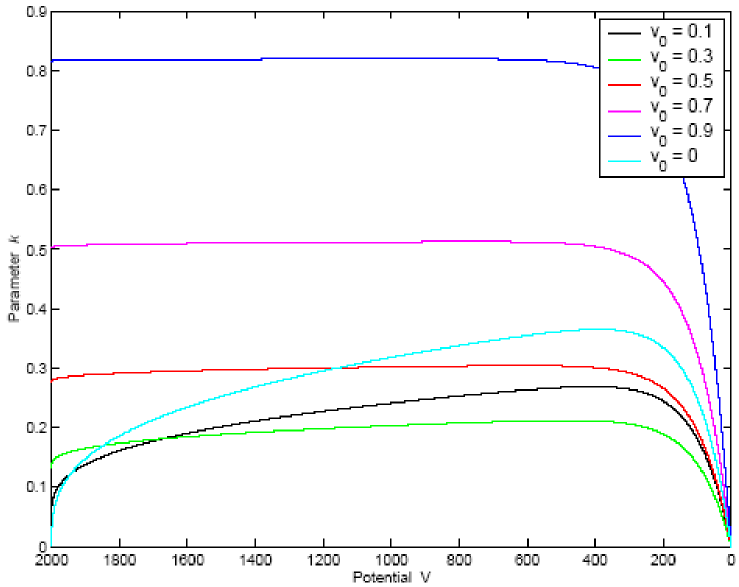

Fig 1. Evolution of parameter *k* for different initial velocities.

The changes introduced so far deal with the amplitude of vector $\mathbf{v}_{desired}$. An initial abrupt acceleration phase is still present. This is the result of an impulsive maneuver to modify the direction of the spacecraft's velocity in order to align it with the desired velocity. The idea is now to introduce a directional control on the spacecraft's velocity vector to allow it to reach the $\mathbf{v}_{desired}$ smoothly. Let γ be the angle between the current and the desired velocity vectors so that:

$$\gamma = \gamma_0 e^{-\beta \tau} \qquad (11)$$

where γ_0 is the initial value of the angle itself, $\tau = t - t_0$ and β is a user defined constant. By combining the amplitude control introduced in Equation 9 and the directional control introduced in Equation 12 the spacecraft current velocity vector gradually moves onto the desired velocity vector, allowing for significant reduction in fuel consumption.

5 Numerical results

To evaluate the performance of the control algorithm a test case is presented. A formation of seven spacecraft is considered; six satellites will form a 50 meters hexagonal formation in the y-z plane, while the last satellite will occupy the central position, offset by 20 meters along the x direction. The initial conditions for both position and velocity are assigned randomly and shown in Table 1, while the final conditions are shown in Table 2.

Table 1. Initial conditions for reconfiguration maneuver.

	x_0 (m)	y_0 (m)	z_0 (m)	$\dot{x}_0$ (m/s)	$\dot{y}_0$ (m/s)	$\dot{z}_0$ (m/s)
Sat 1	– 0.1689	24.4566	12.8785	– 0.0486	– 0.4072	0.0869
Sat 2	– 28.6037	– 23.2053	– 36.6277	– 0.4561	– 0.4647	– 0.4424
Sat 3	14.3492	– 6.0076	– 29.2867	– 0.4728	0.1124	– 0.1324
Sat 4	– 17.9964	43.3380	10.7199	– 0.1873	0.1085	0.1315
Sat 5	46.0099	18.3332	12.9888	– 0.4871	– 0.4842	0.2176
Sat 6	22.6632	– 28.7440	– 12.9523	– 0.1160	– 0.4836	0.1927
Sat 7	– 8.8047	33.9238	7.5148	0.1831	– 0.3099	– 0.4159

Table 2. Final conditions for reconfiguration maneuver.

	x_f (m)	y_f (m)	z_f (m)	$\dot{x}_f$ (m/s)	$\dot{y}_f$ (m/s)	$\dot{z}_f$ (m/s)
Sat 1	100	– 50	0	0	0	0
Sat 2	100	– 25	43.3	0	0	0
Sat 3	100	25	43.3	0	0	0
Sat 4	100	50	0	0	0	0
Sat 5	100	25	– 43.3	0	0	0
Sat 6	100	– 25	– 43.3	0	0	0
Sat 7	80	0	0	0	0	0

As the reconfiguration maneuver is initiated the velocity vector of each spacecraft gradually moves towards the desired velocity vector. The control algorithm provides the collision avoidance capabilities while driving the spacecraft to their goal positions. Figures 2 and 3 show two different views of the reconfiguration maneuver; the hexagonal formation can be clearly seen.

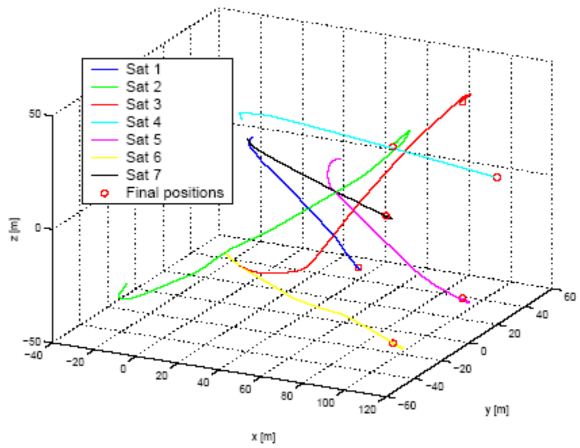

Figure 2. Formation maneuver.

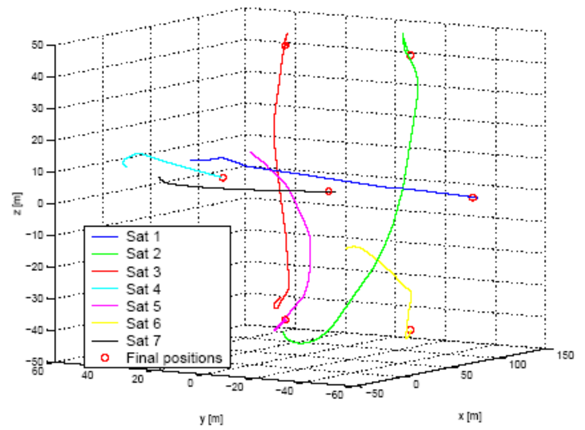

Figure 3. Formation maneuver.

It is interesting now to look at the behaviors of the individual satellites during the maneuver. In Figs. 4-5 we can see the evolution of the potential and its derivate for satellite 1 and satellite 7. It can be seen that for satellite 7 there are no discontinuities in the potential derivative implying a smooth maneuver to the goal position. This is not the case for satellite 1. The potential derivative is highly discontinuous during the initial phase of the maneuver; this is a consequence of the collision avoidance capabilities of the control algorithm.

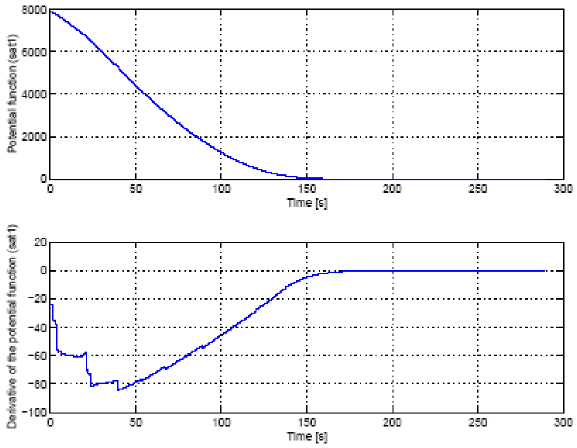

Figure 4. Potential and its derivative for sat 1.

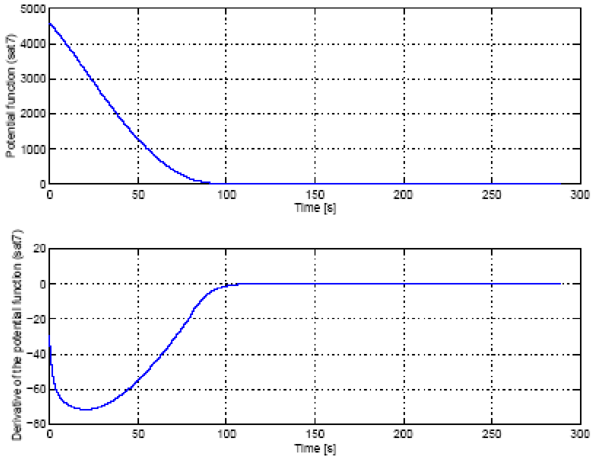

Figure 5. Potential and its derivative for sat 7.

In Fig. 6 we compare the current and desired velocity for satellite 2. Even though the current velocity appears to be altered by the repulsive component of the potential function its overall shape follows quite closely that of the desired velocity.

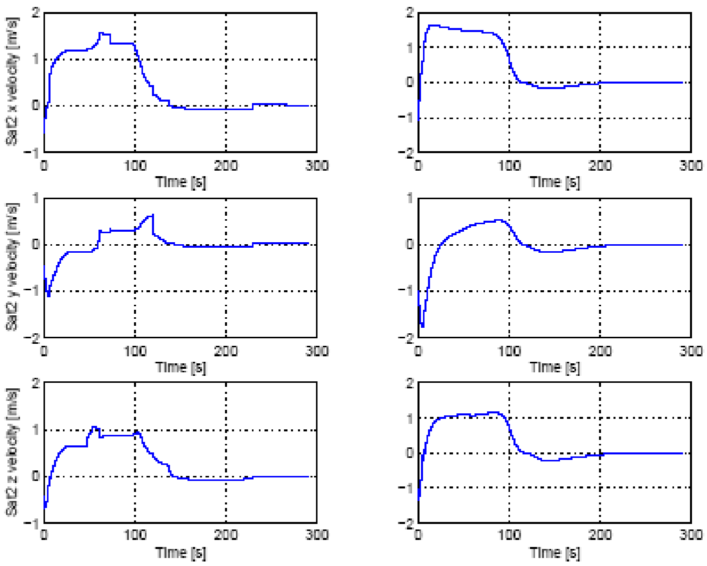

Figure 6. x, y and z components of the current (left) and desired (right) velocity for satellite 2.

In Fig. 7 we can see the evolution of the distance from the target position for the seven satellites. The maneuver is completed successfully in approximately 5 minutes. It should be noted that the majority of the spacecraft overshoot their final position and have to recover from this delaying the end of the maneuver and using more fuel.

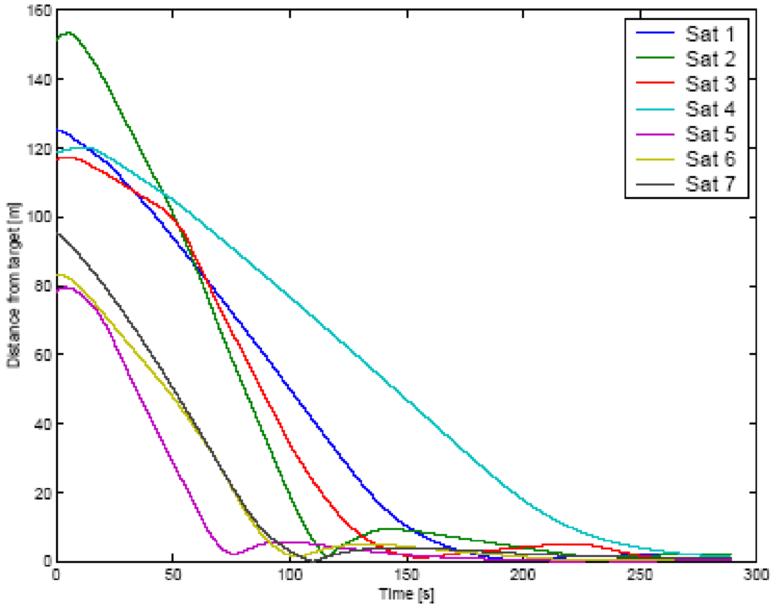

Figure 7. Satellite distance from final position.

Finally in Fig. 8 we can see the spacecraft fuel consumption. If taken individually the consumptions are not high but it can be seen that, as a whole, the formation is not well balanced, meaning that the satellites will experience different operational lifetimes.

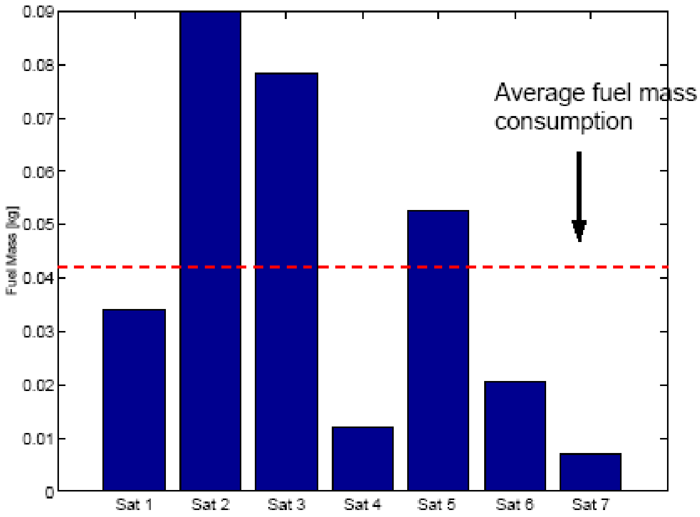

Figure 8. Fuel consumption for the formation.

6 Conclusions

In this paper we have tried to address the problem of improving the fuel consumption of a spacecraft formation controlled via artificial potential functions. We have pointed out a number of issues that influence fuel consumption and suggested a way in which these can be overcome and the performances improved. Further improvements can be achieved by reducing, and ideally canceling out, overshoot displacements. Additionally a more uniform distribution for the fuel consumption throughout the formation should be sought.

7 References

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